

# $\pi NN$ coupling determined beyond the chiral limit

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**Abstract.** Within the conventional QCD sum rules, we calculate the  $\pi NN$  coupling constant,  $g_{\pi N}$ , beyond the chiral limit using two-point correlation function with a pion. For this purpose, we consider the Dirac structure,  $i\gamma_5$ , at  $m_\pi^2$  order in the expansion of the correlator in terms of the pion momentum. For a consistent treatment of the sum rule, we include the linear terms in quark mass as they constitute the same chiral order as  $m_\pi^2$ . In this sum rule, we obtain  $g_{\pi N} = 13.3 \pm 1.2$ , which is very close to the empirical  $\pi NN$  coupling. This demonstrates that going beyond the chiral limit is crucial in determining the coupling.

**PACS.** 13.75.Gx Pion-baryon interactions – 12.38.Lg Other nonperturbative calculations – 11.55.Hx Sum rules

QCD sum rule [1] is a framework which connects hadronic parameters with QCD parameters. In this framework, a correlation function is introduced in terms of interpolating fields constructed from quark and gluon fields. The interpolating field is constructed so that its coupling to the hadron of concern is expected to be strong while its couplings to other higher resonances are hoped to be small. Then the correlator is calculated by Wilson's operator product expansion (OPE) in the deep Euclidean region ( $q^2 = -\infty$ ) and matched with the phenomenological "ansatz" to extract the hadron's information in terms of QCD parameters.

One interesting quantity to be determined is the pion-nucleon coupling constant,  $g_{\pi N}$ . As the coupling is empirically well-known, a successful reproduction of this quantity may provide a solid ground to determine other meson-baryon couplings as well as a better understanding of non-perturbative nature of hadrons. For this direction, the two-point correlation function for the nucleon interpolating field  $J_N$ ,

$$\Pi(q, p) = i \int d^4x e^{iq \cdot x} \langle 0 | T [J_N(x) \bar{J}_N(0)] | \pi(p) \rangle, \quad (1)$$

may be useful and it is often used in calculating  $g_{\pi N}$  [2–5]. Alternative approach is to consider the correlation function without pion but in an external axial field [6]. This provides the nucleon axial charge,  $g_A$ , which can be converted to  $g_{\pi N}$  with the help of the Goldberger-Treiman relation. Our interest in this work is to provide a reasonable value of  $g_{\pi N}$  using (1) because its extension to other meson-baryon couplings seems to be more straightforward.

The correlation function, (1), contains various independent Dirac structures, each of which can be in principle used to calculate  $g_{\pi N}$ . In the recent calculations [4, 5], we have proposed to use the  $\gamma_5 \sigma_{\mu\nu}$  structure in studying  $g_{\pi N}$  as this structure is independent of the pseudoscalar (PS) and pseudovector (PV) coupling schemes employed in the phenomenological side. This sum rule contains very small contribution from the transition  $N \rightarrow N^*$ , and the result is insensitive to the continuum threshold. Therefore, this structure provides a value of  $g_{\pi N}$  independent of the coupling schemes. However, the result from this Dirac structure,  $g_{\pi N} \sim 10$ , is not quite satisfactory. Certainly a further improvement of this sum rule may be needed for future extension to other meson-baryon couplings in SU(3) sector.

Various improvements can be sought for. These may include a question related to the use of Ioffe's nucleon current for the correlator, higher order corrections in the OPE, or corrections associated with the chiral limit. The last possibility for the improvement is interesting because  $g_{\pi N}$  from the  $\gamma_5 \sigma_{\mu\nu}$  sum rule is rather close to the one in the chiral limit than its empirical value. In [4], the calculation is performed beyond the soft-pion limit by taking the leading order of the pion momentum  $p_\mu$ , but for the rest of the correlator the chiral limit,  $p^2 = m_\pi^2 = 0$ , is taken. Thus, it is not clear whether the calculation is done beyond the chiral limit and this may cause the discrepancy with the empirical  $g_{\pi N}$ .

In this paper, we pursue an improvement by presenting a QCD sum rule calculation beyond the chiral limit. Specifically, we consider the Dirac structure,  $i\gamma_5$ , at the order,  $p^2 = m_\pi^2$ . The sum rule for the structure  $i\gamma_5$  is technically less involved when the calculation is done beyond the chiral limit. The OPE side, mainly driven by

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the zeroth and second moments of the twist-3 pion wave function, is relatively well-determined as we will see.

However, since we put the pion on its mass-shell  $p^2 = m_\pi^2$  in this sum rule, one nucleon interacting with the pion will be off the mass-shell, which brings an issue regarding the PS and PV coupling schemes in this  $i\gamma_5$  sum rule. To illustrate the coupling scheme dependence in detail, we use the PS and PV Lagrangians

$$\mathcal{L}_{ps} = g_{\pi N} \bar{\psi} i\gamma_5 \boldsymbol{\tau} \cdot \boldsymbol{\pi} \psi ; \quad \mathcal{L}_{pv} = \frac{g_{\pi N}}{2m} \bar{\psi} \gamma_5 \boldsymbol{\gamma}_\mu \boldsymbol{\tau} \cdot \partial^\mu \boldsymbol{\pi} \psi , \quad (2)$$

in constructing the phenomenological side of the correlator, (1). The two descriptions are equivalent when the nucleons are on their mass-shells. We construct the phenomenological side by inserting the interaction Lagrangians into the correlator and replacing the nucleon interpolating field with its physical one  $J_N \rightarrow \lambda\psi$ . We collect only the part containing the Dirac structure  $i\gamma_5$  and expand it with respect to the pion momentum  $p_\mu$ . Using the PS Lagrangian, we obtain for the  $i\gamma_5$  structure [5],

$$g_{\pi N} \lambda^2 \left[ -\frac{1}{q^2 - m^2} - \frac{p \cdot q}{(q^2 - m^2)^2} + \frac{p^2}{(q^2 - m^2)^2} \right] + \dots \quad (3)$$

Here  $\lambda$  is strength of  $J_N$  to the physical nucleon,  $m$  is nucleon mass.

The first term in the expansion survives in the soft-pion limit. The sum rule for this monopole term is equivalent to the nucleon chiral odd sum rule under the Goldberger-Treiman relation with  $g_A = 1$  [3]. The second term containing  $p \cdot q$  is not the same chiral order as  $m_\pi^2$ . Furthermore, since the two momenta  $p_\mu$  and  $q_\mu$  are independent, this  $p \cdot q$  term should be independent from the third term involving  $p^2$ . Thus in the sum rule at the order  $p^2 = m_\pi^2$  (meaning the pion is on its mass shell), the phenomenological correlator takes the form,

$$m_\pi^2 \frac{g_{\pi N} \lambda^2}{(q^2 - m^2)^2} + \dots \quad (4)$$

The ellipses now indicate the contribution when  $J_N$  couples to higher resonances. This includes the continuum contribution whose spectral density is usually parameterized by a step function with a certain threshold  $S_\pi$ , and single pole terms associated with the transitions,  $N \rightarrow N^*$  [7].

On the other hand, using the PV Lagrangian, the similar recipe yields,

$$\frac{p^2}{2} \frac{g_{\pi N} \lambda^2}{(q^2 - m^2)^2} + \dots \quad (5)$$

This PV correlator contains an additional residue of  $1/2$  compared to the  $p^2$  term in the PS correlator (3). Note that the  $p^2$  term is the first term in this expansion and there is no monopole like the first term in (3). Hence, in the soft-pion limit, this PV correlator vanishes.

Anyway, this factor of  $1/2$  is seemingly strange at the first sight. But we know that this double pole term should

be equivalent to the PS case when the participating nucleons are put on their mass-shells. By putting the nucleons on mass-shells, we immediately obtain the condition,  $p^2 = 2p \cdot q$ . Under this mass-shell condition, the last two terms in (3) are combined and produce the PV correlator (5) as they should be. Therefore the strange factor of  $1/2$  is needed to achieve the equivalence between the PS and PV coupling schemes when the nucleons are on-shell. However, the on-shell condition for both nucleons is not kinematically allowed when we put the pion on its mass-shell  $p^2 = m_\pi^2$ . The three particles cannot be on-shells at the same time. This means that, when the pion is on the mass shell as we study in this work, one nucleon should be off the mass-shell. Therefore, we can not have the condition  $p^2 = 2p \cdot q$  and the last two terms in (3) should not be combined. In other words, they should be treated separately. From this consideration, the PV correlator differs by the factor of 2 at the  $p^2 = m_\pi^2$  order. As the QCD side does not care which coupling scheme is used in the phenomenological side, the coupling constant obtained from the PV scheme will be doubled.

Then a question remains as to which coupling scheme to be used in the construction of the phenomenological side at the  $p^2 = m_\pi^2$  order. Our choice in this work is the PS coupling scheme. To motivate this choice, we consider (1) in the soft-pion limit. For a simple illustration, we replace  $J_N \rightarrow \lambda\psi$ . Using the soft-pion theorem, we replace the correlator (1) by a commutator between the nucleon correlator *without a pion* and the nucleon axial charge. Then it is straightforward to show [8]

$$\begin{aligned} & i\lambda^2 \int d^4x e^{iq \cdot x} \langle 0 | T[\psi(x) \bar{\psi}(0)] | \pi(p_\mu = 0) \rangle \\ &= \frac{\lambda^2}{2f_\pi} \{ \gamma_5, i \int d^4x e^{iq \cdot x} \langle 0 | T[\psi(x) \bar{\psi}(0)] | 0 \rangle \} . \quad (6) \end{aligned}$$

A similar relation is hold in terms of quark degrees of freedom. Thus this relation is supported by the OPE side. Note, the anticommutator in the RHS contains the nucleon propagator. Hence, it is easy to see that the RHS is

$$-\frac{\lambda^2}{f_\pi} \frac{m}{q^2 - m^2} . \quad (7)$$

By comparing with the PS correlator (3) in the soft-pion limit, one can immediately see

$$g_{\pi N} = \frac{m}{f_\pi} , \quad (8)$$

which is nothing else but the Goldberger-Treiman relation with  $g_A = 1$ . More importantly, the RHS of (6) does not vanish in the soft-pion limit. However, this simple fact cannot be recovered from the PV Lagrangian as the PV correlator (5) becomes zero in the soft-pion limit. Thus, the PV coupling scheme seems to have an unpleasant feature. Furthermore, the OPE side for the correlator (1) in its expansion in  $p_\mu$  can be shown to have similar expansion as the PS correlator (3). Therefore, we will use the PS correlator in constructing the phenomenological side.

In future, however, it will be necessary to understand why the PV coupling scheme is not consistent with the OPE.

In the construction of this sum rule, the pion mass,  $m_\pi^2$ , will be kept in both sides of the sum rule. Then, a consistent treatment should be made in the OPE side. Namely, from the Gell-Mann–Oakes–Renner relation,

$$-2m_q \langle \bar{q}q \rangle = m_\pi^2 f_\pi^2, \quad (9)$$

the vanishing limit of the pion mass,  $m_\pi^2 \rightarrow 0$ , is consistent with the chiral limit,  $m_q \rightarrow 0$ . For the sum rule with at the  $m_\pi^2$  order, terms linear in quark mass should be kept in the OPE side as they are in the same chiral order. Clearly, this aspect has been overlooked in our previous calculations [5] and needs to be implemented.

To construct the OPE side, we consider the correlation function with a charged pion,

$$\Pi(q, p) = i \int d^4x e^{iq \cdot x} \langle 0 | T [J_p(x) \bar{J}_n(0)] | \pi^+(p) \rangle. \quad (10)$$

Here  $J_p$  is the proton interpolating field suggested by Ioffe [7],

$$J_p = \epsilon_{abc} [u_a^T C \gamma_\mu u_b] \gamma_5 \gamma^\mu d_c, \quad (11)$$

and the neutron interpolating field  $J_n$  is obtained by replacing  $(u, d) \rightarrow (d, u)$ . In the OPE, we keep the quark-antiquark component of the pion wave function and use the vacuum saturation hypothesis to factor out higher dimensional operators in terms of the pion wave function and the vacuum expectation values.

For the sum rule with the  $i\gamma_5$  structure, we replace the quark-antiquark component of the pion wave function as follows,

$$\begin{aligned} & \langle 0 | u_a^\alpha(x) \bar{d}_{a'}^\beta(0) | \pi^+(p) \rangle \\ & \rightarrow \frac{\delta_{aa'}}{12} (i\gamma_5)^{\alpha\beta} \langle 0 | \bar{d}(0) i\gamma_5 u(x) | \pi^+(p) \rangle. \end{aligned} \quad (12)$$

Other Dirac structures,  $\gamma_5 \gamma_\mu$  and  $\gamma_5 \sigma_{\mu\nu}$ , do not contribute to the  $i\gamma_5$  sum rule at the order  $p^2 = m_\pi^2$ . The matrix element in the left-hand side can be written in terms of the twist-3 pion wave function [9],

$$\begin{aligned} & \langle 0 | \bar{d}(0) i\gamma_5 u(x) | \pi^+(p) \rangle \\ & = -\frac{\sqrt{2} \langle \bar{q}q \rangle}{f_\pi} \int_0^1 du e^{-iup \cdot x} \varphi_p(u). \end{aligned} \quad (13)$$

The second moment of the twist-3 wave function, which is approximately fixed  $\int_0^1 du u^2 \varphi_p(u) = 1/3$  [9], contributes to the sum rule at the  $m_\pi^2$  order. The error for this second moment is very small [9].

Another contribution at this order is obtained by moving a gluon tensor from a quark propagator into the quark-antiquark component. The resulting quark-antiquark component with a gluon tensor can be written in terms the three-particle wave function. Namely, we can write

$$\langle 0 | g_s G_{\mu\nu}^A(0) u_a^\alpha(x) \bar{d}_b^\beta(0) | \pi^+(p) \rangle = B t_{ab}^A (\gamma_5 \sigma_{\mu\nu})^{\alpha\beta}. \quad (14)$$

The color matrices  $t^A$  are related to the Gell-Mann matrices via  $t^A = \lambda^A/2$ . This matrix element should be zero in the soft-pion limit (consistent with the chiral limit). This can be seen easily by using the soft-pion theorem. Multiplying both sides with  $(\gamma_5 \sigma^{\mu\nu})_{\beta\alpha} t_{ba}^A$  leads to

$$B = -\frac{1}{192} \langle 0 | \bar{d}(0) g_s \mathcal{G}_{\mu\nu}(0) \gamma_5 \sigma^{\mu\nu} u(x) | \pi^+(p) \rangle, \quad (15)$$

with  $\mathcal{G}_{\mu\nu} \equiv t^A G_{\mu\nu}^A$ . This matrix element can be directly obtained from [9] [see (36) there]. That is,  $B$  at  $p^2 = m_\pi^2$  order is

$$B \rightarrow \frac{-if_{3\pi} m_\pi^2}{32}, \quad (16)$$

where  $^1 f_{3\pi} = 0.003 \text{ GeV}^2$ .

As we have discussed, terms linear in quark mass should be kept in the OPE for the sum rule at  $m_\pi^2$  order. The quark-mass dependent terms can be obtained by taking the zeroth moment of the twist-3 pion wave function (13), while picking up linear terms in quark-mass from the other part of the correlator <sup>2</sup>. It turns out that the condensates,  $m_q \langle \bar{q}q \rangle$  and  $m_q \langle \bar{q}g_s \sigma \cdot \mathcal{G}q \rangle \equiv m_q m_0^2 \langle \bar{q}q \rangle$ , contribute to the OPE of the  $i\gamma_5$  structure. The Gell-Mann–Oakes–Renner relation can be used to convert  $m_q \langle \bar{q}q \rangle$  to  $-m_\pi^2 f_\pi^2/2$ . Therefore, the quark-mass terms give additional contributions to the sum rule at the  $m_\pi^2$  order.

Collecting all the OPE terms contributing to the  $i\gamma_5$  structure at the  $m_\pi^2$  order, the OPE side (after taking out the isospin factor  $\sqrt{2}$  as well as  $m_\pi^2$  as overall factors) takes the form

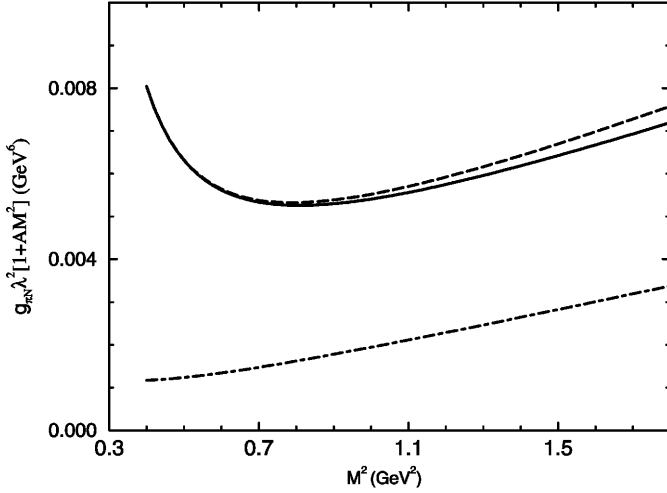
$$\begin{aligned} & \ln(-q^2) \left[ \frac{\langle \bar{q}q \rangle}{12\pi^2 f_\pi} + \frac{3f_{3\pi}}{4\sqrt{2}\pi^2} \right] + f_\pi \langle \bar{q}q \rangle \frac{1}{q^2} \\ & + \frac{1}{72f_\pi} \langle \bar{q}q \rangle \left\langle \frac{\alpha_s}{\pi} \mathcal{G}^2 \right\rangle \frac{1}{q^4} - \frac{1}{3} m_0^2 f_\pi \langle \bar{q}q \rangle \frac{1}{q^4} \end{aligned} \quad (17)$$

Note, we use the pion decay constant  $f_\pi = 0.093 \text{ GeV}$  here. The second and fourth terms come from the quark-mass dependent terms. These are important in stabilizing the sum rule. The second term in the bracket comes from gluonic contribution combined with the quark-antiquark component, (14). Its contribution is about 4 times smaller than the first term in the bracket. Except for this term, all others contain the quark condensate. This feature provides very stable results when this sum rule combined with the nucleon chiral-odd sum rule.

We now match the OPE with its pseudoscalar phenomenological part, (4). To saturate the correlator with

<sup>1</sup> Its value is uncertain by an error  $\pm 0.0005 \text{ GeV}^2$  depending on the renormalization scale [6]. However, the contribution from (14) is small in our sum rule as we will discuss below. Thus, this error in  $f_{3\pi}$  is negligible in our sum rule.

<sup>2</sup> For a complete quark propagator including the linear order in quark mass, see [10]. Note that gluonic tensor used there has opposite sign of that in [2]. This is just a matter of how one defines the covariant derivative but, in practice, this sign difference should be carefully noted.



**Fig. 1.** The Borel mass dependence of  $g_{\pi N} \lambda^2 [1 + AM^2]$ . The solid line is for  $S_{\pi} = 2.07$  GeV<sup>2</sup> and the dashed line is for  $S_{\pi} = 2.57$  GeV<sup>2</sup>. The two curves differ only by 2% at  $M^2 = 1$  GeV<sup>2</sup>. The dot-dashed line is obtained simply by omitting the quark-mass terms in the OPE, indicating the importance of their inclusion in our sum rule

the low-lying resonance, we take the Borel transformation and obtain,

$$\begin{aligned}
& g_{\pi N} \lambda^2 e^{-m^2/M^2} [1 + AM^2] = \\
& -M^4 E_0(x_{\pi}) \left[ \frac{\langle \bar{q}q \rangle}{12\pi^2 f_{\pi}} + \frac{3f_{3\pi}}{4\sqrt{2}\pi^2} \right] - f_{\pi} \langle \bar{q}q \rangle M^2 \\
& + \frac{1}{72f_{\pi}} \langle \bar{q}q \rangle \left\langle \frac{\alpha_s}{\pi} \mathcal{G}^2 \right\rangle - \frac{1}{3} m_0^2 f_{\pi} \langle \bar{q}q \rangle. \quad (18)
\end{aligned}$$

The contribution from  $N \rightarrow N^*$  [7] is denoted by the unknown constant,  $A$ . The continuum contribution is included in the factor,  $E_n(x_{\pi} \equiv S_{\pi}/M^2) = 1 - (1 + x_{\pi} + \dots + x_{\pi}^n/n!)e^{-x_{\pi}}$  where  $S_{\pi}$  is the continuum threshold, taken to be 2.07 GeV<sup>2</sup> corresponding to the Roper resonance. In our analysis, we take standard values for the QCD parameters,

$$\begin{aligned}
& \langle \bar{q}q \rangle = -(0.23 \text{ GeV})^3; \\
& \left\langle \frac{\alpha_s}{\pi} \mathcal{G}^2 \right\rangle = (0.33 \text{ GeV})^4; \\
& m_0^2 = 0.8 \text{ GeV}^2. \quad (19)
\end{aligned}$$

The OPE structure of our sum rule (18) looks similar to the  $\gamma_5 \sigma_{\mu\nu}$  sum rule [4]. Contributions from the terms containing  $f_{3\pi}$  and the gluon condensate are not so large in this sum rule. All others, which are important in our sum rule, add up as they have the same sign. However, in the  $\gamma_5 \sigma_{\mu\nu}$  sum rule [4], the term containing  $m_0^2$  has the opposite sign from the other important terms. The OPE strength of (18) is larger than the one in [4], reducing the role of the  $m_0^2$  term in this sum rule. Therefore, the error due to  $m_0^2$  will be reduced as we will see.

In Fig. 1, we plot  $g_{\pi N} \lambda^2 [1 + AM^2]$  as a function of the Borel mass  $M^2$ . Also shown in the dot-dashed line is when the quark-mass terms are omitted from the OPE.

It clearly shows the importance of the quark-mass dependent terms, justifying their inclusion in our sum rule. To see the sensitivity to the continuum threshold, we also plot the curve with  $S_{\pi} = 2.57$  GeV<sup>2</sup>, which yields the dashed line very close to the solid one plotted with  $S_{\pi} = 2.07$  GeV<sup>2</sup>. The two curves differ only by 2% at  $M^2 = 1$  GeV<sup>2</sup>, indicating that our sum rule is insensitive to the continuum threshold. The highest dimensional term in the OPE contributes appreciably for  $M^2 \leq 0.6$  GeV<sup>2</sup>, more than 20% of the total OPE. Thus, the truncated OPE will be reliable for  $M^2 \geq 0.6$  GeV<sup>2</sup>. The slope of the curve for  $M^2 \geq 0.6$  GeV<sup>2</sup> is small, indicating that the unknown single pole term denoted by  $A$  is small.

To eliminate the dependence on the unknown strength  $\lambda$  in our sum rule, we divide (18) by the nucleon chiral-odd sum rule and obtain,

$$\begin{aligned}
& \frac{g_{\pi N}}{m} [1 + AM^2] = \\
& \left\{ M^4 E_0(x_{\pi}) \left[ \frac{1}{3f_{\pi}} + \frac{3f_{3\pi}}{\langle \bar{q}q \rangle \sqrt{2}} \right] + 4\pi^2 f_{\pi} M^2 \right. \\
& \left. - \frac{\pi^2}{18f_{\pi}} \left\langle \frac{\alpha_s}{\pi} \mathcal{G}^2 \right\rangle + \frac{4\pi^2}{3} m_0^2 f_{\pi} \right\} \\
& \times \left\{ M^4 E_1(x_N) - \frac{\pi^2}{6} \left\langle \frac{\alpha_s}{\pi} \mathcal{G}^2 \right\rangle \right\}^{-1}, \quad (20)
\end{aligned}$$

where  $x_N = S_N/M^2$  with  $S_N$  being the continuum threshold for the nucleon sum rule. Note that the dependence on the quark condensate has been mostly canceled in the ratio, leaving a slight dependence in the term  $f_{3\pi}$ . Additional source of the uncertainty associated with the gluon condensate is also very small as it is canceled in the ratio. One important uncertainty comes from the parameter  $m_0^2$ , which however appears only in the highest dimensional OPE. Thus, its contribution will be suppressed in the Borel window chosen. The error from the QCD parameters is estimated numerically and it is about  $\pm 1.2$  in determining  $g_{\pi N}$ . For the continuum threshold in the nucleon sum rule, we take  $S_N = S_{\pi}$ . This choice is made because at the chiral limit the  $i\gamma_5$  sum rule is equivalent to the nucleon chiral-odd sum rule; these two are related by a chiral rotation. This equivalence provides the Goldberger-Treiman relation with  $g_A = 1$  [3]. This choice for the continuum is also supported from modeling higher resonance contributions to the correlator based on effective models [5]. We determine  $g_{\pi N}$  and  $A$  by fitting the RHS with a straight line within the appropriately chosen Borel window. The dependence on the Borel mass is mainly driven by the nucleon sum rule. The maximum Borel mass is determined by restricting the the continuum contribution from the nucleon sum rule while the minimum Borel mass is obtained by restricting the highest OPE term from the  $\pi NN$  sum rule. These gives the common window of the two sum rules,  $0.65 \leq M^2 \leq 1.24$ . By fitting the RHS with a straight line within this window, we obtain  $g_{\pi N} = 13.3 \pm 1.2$ , where the quoted error comes from the QCD parameters. This is remarkably close to its empirical value of 13.4.

In getting this result, we emphasize that it is essential to go beyond the chiral limit. The quark-mass terms,

included as a consistent chiral counting, are important in stabilizing the sum rule and producing the empirical  $g_{\pi N}$ . This may provide a solid ground for extending to other meson-baryon couplings. The predictive power of QCD sum rules can be substantially enhanced. One application of our sum rule to the  $\eta NN$  is in progress [11].

In summary, we have developed a QCD sum rule for  $\pi NN$  coupling beyond the chiral limit for the first time. The  $i\gamma_5$  Dirac structure at the order  $p^2 = m_\pi^2$  is considered in this sum rule. In the phenomenological side, we have used the pseudoscalar coupling scheme as it seems consistent with the OPE in the soft-pion limit as well as beyond the soft-pion limit. The quark-mass dependent terms are combined into the sum rule as they are the same chiral order as  $m_\pi^2$ . They are very important in this sum rule. A remarkable agreement with the empirical value of  $g_{\pi N}$  was obtained with very small errors.

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